How Gambling in the 17th Century has shaped insurance markets in the 21st century

A short history of probability theory and its ratifications in today's world.

In the 21st century, the insurance market is a pillar in society, providing a safety net for millions while employing over 111 thousand people directly in the UK[1] and approximately 2.69 million people employees in the USA.[2] Insurance dates back to the ancient world, in fact, the first recorded forms of insurance were practiced by Babylonian and traders in as early as 3000 BC where robbed merchants would receive compensation from the governors of the province where the robbery occurred, arguably an example of state insurance .[3][4] However, before the seventeenth century, it was widely agreed that it was impossible to predict an event mathematically, or to assign a value to the likelihood of an event occurring, be it a ship sinking, a thunderstorm or even rolling a dice. Instead, scholars concurred that future events were simply decided by God, and therefore it was inconceivable to even attempt to quantify the likelihood of that event occurring.[5]

Therefore, all prior forms of insurance were in truth, a form of blind gambling. Thus, the discovery of a method of calculating the probability of a given future event occurring was not only a monumental shift in thinking, but was a landmark discovery, in that countless aspects of life today relies, to a greater or lesser extent, on these fundamental calculations. In particular it is evident that probability theory underpins the insurance industry, and the key following breakthroughs were crucial in modernising the insurance business from a guessing game among merchants to the precise science that allows the rest of society to function, using it as a foundation.

---

[1] Statista, "Insurance industry in the United Kingdom - Statistics & Facts,"

https://www.statista.com/topics/4511/insurance-industry-uk/#:~:text=The%20insurance%20industry%20of%20the,any%20sector%20within%20the%20country. [accessed June 10, 2020].

[2] Jennifer Rudden, "Number of employees in the insurance industry in the United States from 1960 to 2018", *Statista*, https://www.statista.com/statistics/194233/aggregate-number-of-insurance-employees-in-the-us/#:~:text=Number%20of%20employees%20in%20the%20insurance%20industry%20in%20the%20U.S.%201960%2D2018&text=In%202018%2C%20there%20were%20approximately,sector%20in%20the%20United%20States.&text=Employment%20within%20the%20insurance%20industry%20has%20shown%20significant%20growth%20since%201960. [accessed June 10, 2020].

[3] Vaughan, E.J. and Vaughan, T., 2007. *Fundamentals of risk and insurance*. John Wiley & Sons. pp. 74

[4] Thoyts, R., 2010. *Insurance theory and practice*. Routledge. pp.101

[5] Devlin, Keith. *The unfinished game: Pascal, Fermat, and the seventeenth-century letter that made the world modern*. Basic Books, 2010. pp.6

Before discussing the fathers of probability, it is fitting to briefly mention the first man to foray into the uncharted waters of probability theory, Girolamo Cardano (1501-1576), a physician, mathematician and avid gambler from Milan. In addition to his contributions to algebra and mechanics, Cardano wrote “Liber de Ludo Aleae" (“Book on Games of Chance”) [6] in 1563, in which he produced the first systematic study of probability regarding dice games. There is still a debate among scholars over whether Cardano wrote the book to assist fellow gamblers, to account for his own gambling addiction, or simply to quench his mathematical curiosity. Nonetheless, by studying dice games, Cardano provides a solid, rudimentary understanding of probability "as a ratio of favourable to total causes (or, outcomes)” [7]He applied this to the throwing of a die, and correctly calculated that the circuit (equal likely cases) is six, and thus, when throwing three dice, there are 216 equally likely cases. Finally, he also grasped the formula p^n when an experiment is repeated *n* times independently.

A good example of this is Cardano’s experiment in which he examines three repetitions of throws of three dice:

He explains that the probability that at least one ‘three’ will be thrown in a single throw is 91/216 as follows:

The total number of outcomes = 6 x 6 x 6 = 216

The number of outcomes without a ‘three’ = 5 x 5 x 5 = 125

∴ the number of outcomes including a ‘three’ = 216-125 = 91 wins. [8]

As there are three repetitions, Cardano reasoned that the probability that the event will occur in each of the repetitions is equal to $(91/216)^3 = 0.0748$.

Although this is very simple for a modern statistician, this an extremely advanced calculation for its time, which demonstrates that Cardano was potentially the first man to appreciate that there is a definitive theory for chance. However, despite finishing his book by 1564, Liber de Ludo Alae was not published until 1663, by which time these concepts were both documented and well

[6] O’connor, J.J. and Robertson, E.F., 2007. Girolamo Cardano. *MacTutor History of Mathematics Archive*.
[7] Williams, Lambert, “Cardano and the Gambler’s Habitus,” Studies of History and Philosophy of Science 36 (2005): 26.
[8] Stigler, S.M., 2015. Is probability easier now than in 1560? *Significance*, *12*(6), pp.42-43.

known to mathematicians across Europe, so unfortunately, Cardano's work had little influence on his successors in the field, despite premature breakthroughs.

The widely regarded fathers of probability, Blaise Pascal (1623-1662) and Pierre de Fermat (1601-165) commenced their correspondence in 1654 when Chevalier de Mere, a French nobleman, posed a classic problem to Pascal: the 'Problem of Points,' otherwise known as the 'Problem of Division of the Stakes' Prior to this dozens of mathematicians had tried and failed to successfully solve the problem, most notably Luca Pacioli (the 'Father of Accounting') in 1494 and Niccolò Tartaglia in 1556 (solver of cubic equations and the first to apply maths to the paths of cannonballs, otherwise known as ballistics), thus Pascal reached out to his friend, the prominent mathematician Pierre de Fermat to assist him, and, in the ensuing letters, the pair solved the problem while laying the groundwork for concepts that are still fundamental to probability theory today.[9]

The problem of Points is as follows:

Two players, call them player A and player B, place equal bets in a game of chance where they are tossing a fair coin. They agree in advance that the first player to win n rounds wins the pot. However, if the game is interrupted before the game is won how should they divide the pot fairly? Suppose player A and player B have a and b points respectively.

Luca Pacioli suggested splitting the stakes in the ratio of the current score. This would result in player A receiving $\frac{a}{a+b}$. parts of the pot while B receives $\frac{b}{a+b}$. However, the aforementioned Cardano was keen to highlight his errors that "even a child should recognise." Cardano spotted that if only one round were played, the entire pot would go to the player with one point regardless of how many rounds were yet to be player. Therefore, Cardano argued that the number of rounds required for each player to win the contest should decide the ratio that the stakes are divided. Although this concept assisted others find the solution, Cardano failed to establish the correct answer himself. The next to attempt to solve the problem was Niccolò Tartaglia[10] who again criticized Pacioli's method while proposing that a better solution is that assuming player A has more rounds won than B, player A should keep his own stake while taking the fraction $\frac{a-b}{n}$ of player B's bets. This would result in a division of the total pot of $n + a$

[9] Burton, D.M., 1985. The history of mathematics: An introduction. *Group*, *3*(3). p.454
[10] Bruneau, O., Heering, P., Grapí, P., Esteve, M.R.M., Laubé, S. and de Vittori, T. eds., 2012. *Innovative Methods for Science Education: History of Science, ICT and Inquiry Based Science Teaching*. Frank & Timme GmbH. mnpg.347

– *b* to *n* - *a* + *b.* Although this is slightly more accurate than Pacioli's solution, it has exactly the same flaws, which led Pacioli to concede that he believed that the problem was impossible to solve in a way that would be fair for both players.[11]

Having stumped some of the greatest mathematical minds ever, it seemed that Tartaglia might be correct and that no solution could be formed. However, in their letters, Fermat, in particular, quickly began building on Cardano's idea to form the following argument:

If r is the number of rounds player A needs to win and s is the number of rounds player B needs to win, the game will be over after a maximum of $r + s - 1$ rounds. Using Cardano's formula for repeated rounds (p^n), Fermat reasoned that these unfinished rounds have 2^{r+s-1} possible outcomes and was therefore able to simply write down the possible outcomes and calculate a ratio by counting how many of the rounds are won by each player.

Fermat realised that in some of these outcomes, the game would already have already been decided before $r + s - 1$ rounds had been played, however, he correctly argued it does no harm to imagine the players continued playing out all the rounds once the game had been won. This solution created the notion of expected value. Clearly, Fermat's solution, despite even today being considered fair and correct, rapidly becomes inconvenient when having to examine much longer games with many more outcomes. Pascal, despite an obvious reluctance to buy into Fermat's theory, added to this model, using his triangle, by expressing it as a more complex summation formula.

Although the maths does not appear to be complex, the idea of hanging numbers into the future was so revolutionary, that even Pascal seemed to deeply struggle with the concept. The fact that it seems so simple, and so obvious, reflects the vast magnitude of this cognitive shift. Once this controversial problem had been solved, and the notions of expected values had been conceived, the ideas that both Pascal and Fermat's different solutions had formed rapidly captured the minds of intellectuals across Europe.

Within three years, Christiaan Huygens had produced the first mathematical study on probability. The study was published in a sixteen-page pamphlet entitled 'De Ratiociniis in ludo aleae' and

---

[11] Hald, A., 2005. *A history of probability and statistics and their applications before 1750* (Vol. 574). John Wiley & Sons. pg. 35

was, used as the standard text on probability for the next 50 years.'[12] Huygens was quick to appreciate and recognise that his work was built upon Pascal and Fermat's contribution, stating in a letter that "for some time some of the best mathematicians of France have occupied themselves with this kind of calculus so that no one should attribute to me the honour of the first invention"[13]Despite Huygens modesty, it is clear that his publication had ground-breaking effects. Not only did he sort out Fermat's and Pascal's fairly untidy concepts into a coherent, systematic method, but he recognised the gravity of their findings, supplemented them and made it explicit how significant they were, stating, "I would like to believe, that if someone studies these things a little more closely, then he will almost certainly come to the conclusion that it is not just a game which has been treated here, but that the principles and the foundations are laid of a very nice and very deep speculation." [14]

Huygens was indeed correct as shortly after entitled 'De Ratiociniis in ludo aleae' was published, John Graunt (1620-1674), with the assistance of his friend, Sir William Petty, created a large survey of those who died in London, noting basic information about the deceased in London[15], and published his findings in his short pamphlet 'Natural and Political Observations Made Upon the Bills of Mortality' [16] [17] Huygens spotted the opportunity to take his probability theory out of the gambling parlours and apply them to this data, updating Graunt's study. Later, in 1693, Edmund Halley, the celebrated Astronomer, improved Huygens's model using much more accurate mortality statistics: parish records of a Polish city, Breslau, where little to no migration occurred, to produce Halley's life table- a detailed study giving the odds of not dying at a given age. This paper became a cornerstone in actuarial mathematics.

The final phases of applying probability into insurance occurred within a lifetime of Pascals discovery. Nikolaus Bernoulli implemented probability theory into the legal problems in 1709 [18]while his uncle, Jakoub Bernoulli (1654-1705) produced a study of probability entitled 'Ars Conjectandi' in which he first used the word 'probability', and proved his law of large numbers - he argued that a large sample is a solid reflection of the entire population.[19] Finally, in 1733 Abraham de Moivre (1667-1754) developed the bell curve as a "doctrine of chances"[20] which,

---

[12] Ibid p.68

[13] Otten, N., *Huygens and The Value of all Chances in Games of Fortune.* Pp. *3*
[14] Devlin, *The unfinished game*, pp..98
[15] Timmreck, T.C., 2002. *An introduction to epidemiology*. Jones & Bartlett Learning.pp.33-34
[16] Ferguson, N., 2008. *The ascent of money: A financial history of the world*. Penguin.pp.188
[17] Clark, G. and Clark, G.W., 1999. *Betting on lives: the culture of life insurance in England, 1695-1775*. Manchester University Press. pp115-116
[18] Grattan-Guinness, I. ed., 2005. *Landmark writings in Western mathematics 1640-1940*. Elsevier. pp.103
[19] Hald, A., 2008. *A history of parametric statistical inference from Bernoulli to Fisher, 1713-1935*. Springer Science & Business Media.pp.11
[20] Phillips, D.C. ed., 2014. *Encyclopedia of educational theory and philosophy*. Sage Publications. pp.84

again, could then further help statisticians and mathematicians apply the theory of probability to real-life applications, such as insurance.

Gambling was at the heart of every early discovery relating to probability (with the prominent exception of Pascal's Wager- essentially that humans have little to lose by believing in god), not only because probability is a major part of gambling , but also due to the large, highly competitive, and often very academic nature of the gambling community and the willingness of the community to share ideas, displayed by Pascal and Fermat. As time passed, it became evident that Huygen's evaluation of the importance of Fermat and Pascal's discovery in particular, was indeed very true. The French pair had not merely solved a seemingly trivial gambler's dispute that only occurs in very specific conditions; but unbeknownst to either of them, they had unlocked the gates to a fundamental branch of mathematics that society began to rely on as it developed. The discovery that risk could be precisely calculated revolutionised the implementation of statistics, and calculating that risk is fundamental to many aspects in life, notably the insurance markets. In short, it was not ancient merchants, but gambling mathematicians who were the true forefathers of insurance.

Bibliography

Bruneau, O., Heering, P., Grapí, P., Esteve, M.R.M., Laubé, S. and de Vittori, T. eds., 2012. *Innovative Methods for Science Education: History of Science, ICT and Inquiry Based Science Teaching*. Frank & Timme GmbH.

Burton, D.M., 1985. The history of mathematics: An introduction. *Group*, *3*(3)

Clark, G. and Clark, G.W., 1999. *Betting on lives: the culture of life insurance in England, 1695-1775*. Manchester University Press.

Devlin, Keith. *The unfinished game: Pascal, Fermat, and the seventeenth-century letter that made the world modern*. Basic Books, 2010.

Ferguson, N., 2008. *The ascent of money: A financial history of the world*. Penguin.

Grattan-Guinness, I. ed., 2005. *Landmark writings in Western mathematics 1640-1940*. Elsevier.

Hald, A., 2008. *A history of parametric statistical inference from Bernoulli to Fisher, 1713-1935*. Springer Science & Business Media..

Hald, A., 2005. *A history of probability and statistics and their applications before 1750* (Vol. 574). John Wiley & Sons.

O'connor, J.J. and Robertson, E.F., 2007. Girolamo Cardano. *MacTutor History of Mathematics Archive*.

Otten, N., *Huygens and The Value of all Chances in Games of Fortune.*

Phillips, D.C. ed., 2014. *Encyclopedia of educational theory and philosophy*. Sage Publications.

Statista, "Insurance industry in the United Kingdom - Statistics & Facts," https://www.statista.com/topics/4511/insurance-industry-uk/#:~:text=The%20insurance%20industry%20of%20the,any%20sector%20within%20the%20country [accessed June 10, 2020]

Stigler, S.M., 2015. Is probability easier now than in 1560? *Significance*, *12*(6),

Thoyts, R., 2010. *Insurance theory and practice*. Routledge.

Timmreck, T.C., 2002. *An introduction to epidemiology*. Jones & Bartlett Learning.

Todhunter, I., 1865. *A History of the Mathematical Theory of Probability from the Time of Pascal to that of Laplace*. Macmillan.

Vaughan, E.J. and Vaughan, T., 2007. *Fundamentals of risk and insurance*. John Wiley & Sons.

Williams, L., 2005. Cardano and the gambler's habitus. *Studies in History and Philosophy of Science Part A*, *36*(1).